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On the profile of the liquid wedge underneath a growing vapour bubble and the reversal of the wall heat flux

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Abstract

Combining mass and energy balances, a differential equation for the profile of a liquid wedge underneath a vapour bubble that is growing on a solid surface is derived. It connects the spatial and temporal changes of the film (wedge) thickness with the spatial temperature changes and velocity of liquid at the interface. Specifying particular conditions, the equation reduces to those from the literature. The paper brings further an illustrative explanation of why the wall heat flux in the wedge region may reverse its direction. © 2001 Elsevier Science Ltd. All rights reserved.

1. Introduction

Many of the recent publications have substantially contributed to our better understanding of bubble physics, and in particular those of Welch [1], Wilson et al. [2], and Son et al. [3], all appeared in the last two years. Welch [1] has performed a direct numerical simulation of a single vapour bubble. Assuming a physical system, consisting of a solid wall, a liquid pool and a vapour bubble that is adhering to the wall surface, first to be at equilibrium, he increased jumpwise the system temperature (all three phases). The bubble growth thus initiated develops a temperature field, which allows a heat flow from the liquid to the wall in a certain region of the liquid–wall interface. Although obtained under adiabatic conditions of the whole system and, in addition, at a pinned three-phase-line (TPL, liquid–vapour–solid), Welch's results become essential, if we are to imagine what might happen around a vapour bubble growing on a heating surface. The picture he obtained is directly applicable to nucleate boiling, at least in the case of a low spatio-temporally averaged wall heat flux.

By numerical experiments, Wilson et al. [2] have demonstrated, among other things, that the interface of a two-dimensional (cylindrical) vapour bubble sandwiched between two isothermal superheated parallel plates is convex near the TPL. This line was not pinned on the plate surface, but was allowed to move as the bubble grows. During the whole observation time period, the interface preserves its convex shape in the TPL region. When evaluated in terms of boiling under real conditions, this finding does not seem to be seriously affected by the assumption of the bubble to be cylindrical and the plates to be isothermal. Finally, Son et al. [3] have shown numerically that the bubble detachment occurs via a necking process. Also in this case the constant wall temperature taken for simulation does not materially affect the bubble necking.

The results reported in the above-mentioned papers confirm earlier notions about the bubble behaviour. On the basis of a qualitative reasoning, the author of the present paper has stated [4–8] that

- the vapour–liquid interface of a growing bubble is concave–convex at the TPL,
- the wall heat flux may locally reverse its direction and the heat flows from the liquid to the wall,
- the bubble detachment occurs via necking,
- the vapour rest remaining on the wall after the bubble break-off, exposed to a strong action of the Laplace pressure, may condense. The fate of this vapour rest affects the waiting period of the next bubble.

It has further been concluded that the movement (expansion) of the TPL during the bubble growth is

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Nomenclature

A	area
δA	interfacial area element
c_p	specific heat capacity
h	specific enthalpy
Δh	latent heat
$\dot{m}$	mass flux
$\delta \dot{M}$	mass flow rate
$\vec{q}$	heat flux vector
q^{ξ}, q^{η}	ξ and η heat flux components
$\delta \dot{Q}$	heat flow rate
R, S	mass and heat flow terms arising from wedge slope
T	temperature
t	time
TPL	three-phase-line
u, v	ξ and η velocity components
V	volume
$\vec{w}$	velocity vector

Greek symbols

δ	film thickness
κ	thermal diffusivity
λ	thermal conductivity
η	axial coordinate
ρ	density
ξ	radial coordinate

Subscripts

L	liquid
V	vapour
t	derivation with respect to t
W	at wall surface
δ	at film surface
η	derivation with respect to η
ξ	derivation with respect to ξ

Superscripts

η	component in η direction
ξ	component in ξ direction

preceded by a rapid evaporation of liquid molecules making this line, whereas the contraction of the TPL during the bubble detachment does not allow any reliable determination of its length and the contact angle in a simple way [5,7]. These quantities become fundamental if the bubble detachment is discussed in terms of a force balance, which is sometimes called "Tate's law", see Tate [9], Mitrovic [5], Lubetkin [10]. As it is now believed, a force balance does not adequately mimic the detachment of a capillary body (vapour bubble, liquid droplet), and in this context a further source deserves, at least historically, to be mentioned. Quarter of a millennium ago, about 1748, Boscovich [11] has precisely described the break-off of a hanging droplet via the necking process, [1] which is immediately applicable to a vapour bubble.

While closing this brief literature review, we should remark that Ilyin et al. [12] have observed experimentally a negative wall heat flux during bubble growth. The experiments have been performed with water at atmospheric pressure, at a relatively low wall heat flux of 35.8 kW/m^2, a wall temperature of 110.5°C, and a water bulk temperature of 99.5°C. The authors state:

> ... Secondly, interference fringe pattern in the thin liquid layer beneath the bubble shows the direction of local heat flux not only from the liquid wedge to the bubble, but also from the liquid to the subcooled metal in certain stages of the vapour bubble growth.

An explanation of this phenomenon is not given in their paper. The possible shapes of the isotherms in the region of the heat flux reversal, discussed in [6], are qualitatively in agreement with the ones reported by Welch [1]. Although physically of great consequence and scientifically highly exciting, it is curious that no publication devoted to modelling and numerical treatment of bubble growth does mention the Ilyin et al. results.

The present paper was motivated by the analysis of Wilson et al. [2]. Our main aim is to derive a differential

[1] As Boscovich's [11] notion applies to any capillary body and, in addition, possesses a certain historical value, I cite from 434 of the second edition of the Theoria:

> ... in the case of drops of water hanging suspended; here, as soon as they have increased up to a point where the weight of the whole drop becomes greater than the mutual attractive force of its parts, any greater part is not torn away as a whole, but by degrees, though in a time that is exceedingly short, the drop is attenuated at its upper part, until the neck, which has by now become exceedingly narrow, is finally broken altogether. There were, say, initially, a thousand particles in the surface connecting the hanging drop to the upper part of the water which is left adhering to the body from which the drop was suspended; these a little afterwards become 900, then 800, then 700, & so on, their number being gradually diminished as the sides of the neck approach one another, & its figure is narrowed.

The process of droplet detachment is one of many examples Boscovich has taken to illustrate his notion upon mutual interactions between elementary particles of matter.

expression for the profile of the liquid wedge underneath a growing bubble. In the case of a two-dimensional, infinitely long cylindrical bubble, such an equation is easily deducible from the results of Wilson et al. [2]. Here, we assume a rotationally symmetrical vapour bubble and first derive expressions for the interfacial mass and heat fluxes; then, we combine them to a differential equation that describes the wedge profile. Contrary to Son et al. [3], we do not employ any lubrication hypothesis; this will make it possible to identify further terms that affect the unsteady-state wedge shape. From this point of view, the present paper supplements the analytical considerations reported in [8]; also here, we do not pursue any numerical evaluations. Included in the paper is a brief model discussion that should visualize the origin of the negative wall heat flux during the bubble growth.

2. Mass and energy balances and the wedge profile

2.1. Mass balance and interfacial mass flux

The model adopted is sketched in Fig. 1. Shown is a rotational–symmetrical vapour bubble that is growing on an ideally smooth, superheated wall surface. The black dots represent the TPL; this line borders a circular area, across which the vapour interacts with the wall. Our attention is focussed on an element of the liquid wedge that is sandwiched between the vapour and the wall surface outwards the TPL; the control volume is extended by the element $\mathrm{d}\xi$ along the radial axis of a cylindrical coordinate system, the axial coordinate of which coincides with the axis of bubble symmetry.

The mass balance for the control volume can be written as

$$\frac{\partial}{\partial t}\int_V \rho\,\mathrm{d}V = -\int_A \left(\rho\vec{w},\,\mathrm{d}\vec{A}\right) - \delta\dot{M}_\delta, \tag{1}$$

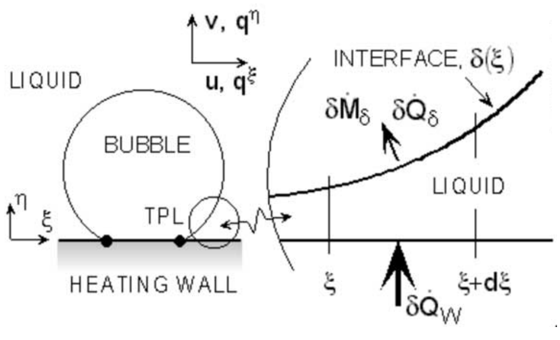

Fig. 1. Physical model illustrating a wedge-shaped liquid region along the rim of the vapour–wall interaction surface.

where t denotes the time, ρ the liquid density, $\vec{w}$ the velocity vector, $\mathrm{d}\vec{A}$ the surface element and $\delta\dot{M}_\delta$ the mass flow rate across the vapour–liquid interface. The surface integral must, therefore, not be applied to the vapour–liquid portion of the control surface.

With $\mathrm{d}V = 2\pi\xi\,\mathrm{d}\xi\,\mathrm{d}\eta$, $\mathrm{d}A = 2\pi\xi\,\mathrm{d}\eta$ and $\rho = \mathrm{const}$, Eq. (1) gives

$$2\pi\rho\left(\delta_t\xi + u_\delta\delta_\xi\xi + \int_0^\delta \frac{\partial}{\partial\xi}(u\xi)\,\mathrm{d}\eta\right)\mathrm{d}\xi = -\delta\dot{M}_\delta + R, \tag{2}$$

where the subscripts t and ξ attached to the film thickness δ indicate its partial derivatives with respect to these variables, and the index δ refers to the vapour–liquid interface. The quantity R stands for further terms arising from the fact that the film surface is not parallel to the plane $\eta = 0$, but is sloped. For simplicity, however, this quantity is considered to be small and omitted within the present paper. The error thus introduced is negligible in the most wedge region, but not necessarily at the TPL with a large surface curvature [8].

Involving the equation of continuity at $\rho = \mathrm{const}$

$$\xi\frac{\partial v}{\partial\eta} + \frac{\partial}{\partial\xi}(\xi u) = 0 \tag{3}$$

and taking the velocity u to depend parametrically on the film thickness δ, the integral in Eq. (2) can be solved to give

$$\delta\dot{M}_\delta = -2\pi\rho(\delta_t + u_\delta\delta_\xi - v_\delta)\xi\,\mathrm{d}\xi. \tag{4}$$

Defining a mass flux $\dot{m}_\delta$ at the interface by

$$\dot{m}_\delta\delta A_\delta = \delta\dot{M}_\delta, \tag{5}$$

where δA_δ is the size of the interfacial area, $\delta A_\delta = 2\pi(1+\delta_\xi^2)^{1/2}\xi\,\mathrm{d}\xi$, we get

$$\dot{m}_\delta = -\rho(\delta_t + u_\delta\delta_\xi - v_\delta)\left(1+\delta_\xi^2\right)^{-1/2} \tag{6a}$$

or

$$\dot{m}_\delta = -\rho\left(\frac{\partial\delta}{\partial t} + u_\delta\frac{\partial\delta}{\partial\xi} - v_\delta\right)\left(1 + (\partial\delta/\partial\xi)^2\right)^{-1/2}. \tag{6b}$$

This equation has the same shape as the one reported by Wilson et al. [2] for a cylindrical vapour bubble, see also Burelbach et al. [13].

For a vapour–liquid interface parallel to the wall surface, we have $\delta_\xi = \partial\delta/\partial\xi = 0$, and Eq. (6a) reduces to $\dot{m}_\delta = -\rho(\delta_t - v_\delta)$. For an interface orthogonal to the wall surface, $\delta_\xi = \partial\delta/\partial\xi \to \infty$, resulting in $\dot{m}_\delta = -\rho u_\delta$, if both the axial velocity and the local change of the film thickness are finite.

2.2. Energy balance and interfacial heat flux

In order to derive an expression for the interfacial heat flux q_δ, we will start from the energy balance. Referring to Fig. 1, we may write

$$\frac{\partial}{\partial t}\int_V \rho c_p T\, \mathrm{d}V = -\int_A \left(\vec{q}, \mathrm{d}\vec{A}\right) - \delta\dot{Q}_\delta + \delta\dot{Q}_W, \tag{7}$$

where $\vec{q}$ is the energy flux vector, T is the temperature, and $\delta\dot{Q}_\delta$ and $\delta\dot{Q}_W$ are the heat flow rates at the interface and the wall surface, respectively. Note that the enthalpy and temperature in the reference state are taken to be zero.

Proceeding as above for the mass flux $\dot{m}_\delta$, we obtain

$$2\pi\left(\rho c_p T_\delta \delta_t \xi + q^\xi \delta_\xi \xi + \int_0^\delta \left(\rho c_p \xi \frac{\partial T}{\partial t} + \frac{\partial}{\partial \xi}\left(q^\xi \xi\right)\right) \mathrm{d}\eta\right) \mathrm{d}\xi = -\delta\dot{Q}_\delta + \delta\dot{Q}_W + S, \tag{8}$$

where S, like R in Eq. (2), arises from the slope of the film surface. We also neglect the quantity S in the present discussion.

Since the physical properties are constant, the local energy balance may be written as

$$\rho c_p \xi \frac{\partial T}{\partial t} + \frac{\partial}{\partial \xi}\left(q^\xi \xi\right) = -\frac{\partial}{\partial \eta}\left(q^\eta \xi\right), \tag{9}$$

so that for the heat flux q^η considered to be an implicit function of δ the integral in Eq. (8) can be solved to give

$$2\pi\left(\rho c_p T_\delta \delta_t \xi + q_\delta^\xi \delta_\xi - q_\delta^\eta + q_W^\eta\right)\xi\, \mathrm{d}\xi = -\delta\dot{Q}_\delta + \delta\dot{Q}_W. \tag{10}$$

In Eqs. (8)–(10), the superscripts η and ξ refer to the radial and axial directions of the coordinate system, respectively, and the index W to the wall surface.

Introducing an interfacial energy flux q_δ by

$$q_\delta \delta A_\delta = \delta\dot{Q}_\delta, \tag{11}$$

and considering the relation $\delta\dot{Q}_W = 2\pi q_W^\eta \xi\, \mathrm{d}\xi$, we get the expression

$$q_\delta = -\left(\rho c_p T_\delta \delta_t + q_\delta^\xi \delta_\xi - q_\delta^\eta\right)\left(1 + \delta_\xi^2\right)^{-1/2}, \tag{12}$$

which is explicitly independent of the heat flux on the wall. This equation allows an interesting conclusion, namely, the capacity term in the parenthesis results in a larger heat flux q_δ when at $\delta_t = \partial\delta/\partial t < 0$ the saturation temperature (pressure) is increased. This relationship explains at least partly why the heat flux in nucleate boiling sensitively increases with increasing pressure.

2.3. Differential expression for the wedge profile

To derive a differential equation for the film profile, we will start with Eq. (12). The components q_δ^ξ and q_δ^η of the interfacial energy flux vector in this equation are to be obtained from

$$q_\delta^\xi = -\lambda \frac{\partial T}{\partial \xi} + \rho u_\delta h_L = -\lambda T_\xi + \rho u_\delta h_L, \tag{13}$$

$$q_\delta^\eta = -\lambda \frac{\partial T}{\partial \eta} + \rho v_\delta h_L = -\lambda T_\eta + \rho v_\delta h_L, \tag{14}$$

h_L being the liquid enthalpy ($h_L = c_p T$), so that

$$q_\delta = -\left(\rho(\delta_t + u_\delta \delta_\xi - v_\delta)h_L - \lambda\left(T_\xi \delta_\xi - T_\eta\right)\right)\left(1 + \delta_\xi^2\right)^{-1/2}. \tag{15}$$

Insisting on a continuity of the energy flux across the vapour–liquid interface, we may write

$$q_\delta = \dot{m}_\delta h_V \tag{16}$$

where h_V is the enthalpy of vapour leaving the interface. Note that Eq. (16) disregards a possible temperature gradient in the vapour. This is allowable in most cases, but requires, in a strict sense, an isothermality of the interface. We will return to this question further below.

Combining Eqs. (6a), (15) and (16) and setting $\Delta h = h_V - h_L$, gives

$$\delta_t + u_\delta \delta_\xi - v_\delta + \left(T_\xi \delta_\xi - T_\eta\right)\frac{\lambda}{\rho \Delta h} = 0, \tag{17a}$$

or

$$\frac{\partial \delta}{\partial t} + u_\delta \frac{\partial \delta}{\partial \xi} - v_\delta + \left(\frac{\partial T}{\partial \xi}\frac{\partial \delta}{\partial \xi} - \frac{\partial T}{\partial \eta}\right)\frac{\lambda}{\rho \Delta h} = 0, \tag{17b}$$

which describes the film profile and specifies the spatio-temporal conditions at the interface to be satisfied by the flow and temperature fields within the liquid wedge. Note that also the boundary conditions demanded by the momentum equation are implicitly included in Eqs. (17a) and (17b) via the interfacial velocity components u_δ and v_δ. For a detailed analysis of the later conditions, the reader is referred to the article by Wilson et al. [2].

Eqs. (17a) and (17b) allows constructions of several model cases. The simplest one is deduced by assuming the liquid in the wedge to be at rest, $u_\delta = v_\delta = 0$. Then,

$$\frac{\partial \delta}{\partial t} + \left(\frac{\partial T}{\partial \xi}\frac{\partial \delta}{\partial \xi} - \frac{\partial T}{\partial \eta}\right)\frac{\lambda}{\rho \Delta h} = 0, \tag{18}$$

which basically belongs to the family of the so-called Stefan problems; in this case, the movement of the interface occurs only by phase change.

For a steady-state case, Eqs. (17a) and (17b) simplifies to

$$\left(\left(u_\delta + \frac{\lambda}{\rho \Delta h}\frac{\partial T}{\partial \xi}\right)\frac{\partial \delta}{\partial \xi} - v_\delta\right)\rho \Delta h - \lambda \frac{\partial T}{\partial \eta} = 0, \tag{19}$$

from which for the temperature T assumed to be independent of ξ, we obtain

$$\left(u_\delta \frac{\partial \delta}{\partial \xi} - v_\delta\right)\rho \Delta h - \lambda \frac{\partial T}{\partial \eta} = 0. \tag{20}$$

Eq. (19) is immediately applicable to evaporation at free liquid menisci frequently occurring e.g., in grooves of heat pipes, where the profile of the vapour–liquid interface is "frozen" in time. However, as a rule, Eq. (19) is not used there, but the one resting on the so-called lubrication theory. In this context the reader is referred to the recent review paper by Wayner [14] and references therein. Some bubble growth models from the literature use even the simpler Eq. (20). On the basis of this equation it has been reasoned in [8] that the liquid wedge at the TPL, where for no sliding conditions at the wall surface both velocity components became zero, is concave–convex. This is in agreement with the numerical results by Wilson et al. [2].

To deduce from Eqs. (17a) and (17b) the expression reported by Son et al. [3] (Eq. (5) in [3]), one has to take $\partial T/\partial \eta$ to be constant over the film thickness ($\partial T/\partial \eta = -q/\lambda$, q being the wall heat flux) and to set $\partial T/\partial \xi = 0$. In addition, the term ($v_\delta - u_\delta \partial\delta/\partial\xi$) has to be identified as the velocity normal to the vapour–liquid interface, which is allowable for relatively flat films ($\partial\delta/\partial\xi \to 0$) only.

Prior to proceeding to discuss the origin of a negative wall heat flux in the region of the TPL of a growing bubble, let us make two remarks. First, the expressions (6a), (15) and (17a) and (17b) for the interfacial mass and heat fluxes, are also valid for a two-dimensional (Cartesian) wedge, in which case ξ and η become the Cartesian coordinates. The results of Wilson et al. [2] are, therefore, applicable to vapour bubbles having the shape depicted in Fig. 1. Second, all the above considerations are confined to the liquid phase only. As a consequence, we get from Eqs. (17a) and (17b) at the thermodynamic critical point ($\Delta h = 0$) of the fluid the expression

$$\frac{\partial T}{\partial \xi}\frac{\partial \delta}{\partial \xi} - \frac{\partial T}{\partial \eta} = 0, \tag{21}$$

which – within our model – states that a control surface replacing the former vapour–liquid interface is adiabatic at this point. By contrast, Eq. (15) allows correctly a heat flow across such surface.

In order to derive a more general equation it suffices to extend the control volume in Fig. 1 into the vapour phase. Then, from a mass balance, we obtain

$$\Delta\rho \frac{\partial \delta}{\partial t} + (\rho_L u_{L\delta} - \rho_V u_{V\delta})\frac{\partial \delta}{\partial \xi} - (\rho_L v_{L\delta} - \rho_V v_{V\delta}) = 0, \tag{22}$$

whereas an energy balance gives

$$\frac{\partial \delta}{\partial t} + u_{L\delta}\frac{\partial \delta}{\partial \xi} - v_{L\delta} + \left(\left(\frac{\partial T_L}{\partial \xi} - \frac{\lambda_V}{\lambda_L}\frac{\partial T_V}{\partial \xi}\right)\frac{\partial \delta}{\partial \xi} - \left(\frac{\partial T_L}{\partial \eta} - \frac{\lambda_V}{\lambda_L}\frac{\partial T_V}{\partial \eta}\right)\right)\frac{\lambda_L}{\rho_L \Delta h} = 0. \tag{23}$$

The indices L and V added to the variables in Eqs. (22) and (23) should discriminate between the two phases. These equations are identically satisfied at the thermodynamic critical point. Note that in Eq. (23), like in Eqs. (17a) and (17b), the temperature derivatives are to be obtained at the vapour–liquid interface. For a steady-state evaporation at a planar liquid film ($\partial\delta/\partial\xi = 0$), Eq. (22) expresses the continuity of the mass flux across the interface, whereas by omitting the Fourier terms in the vapour phase, Eq. (23) simplifies to Eqs. (17a) and (17b).

3. Reversal of the wall heat flux

Several publications devoted to bubble growth either adopt steady-state model equations or assume an isothermal wall surface. Models with such restrictions, by no means useful in many respects, do not detect some details, which are important for obtaining a more complete picture of bubble physics. For instance, the reversal of the wall heat flux remains inaccessible by these models.

As mentioned above, Ilyin et al. [12] have shown experimentally and Welch [1] numerically that the wall heat flux during a bubble cycle is not uniformly directed in the bubble influence region, but a plausible explanation of this somewhat puzzling phenomenon has not been offered yet. The only exception is an earlier try by the present author [6].

To illustrate the reversal origin of the wall heat flux, that is, the heat flow from the liquid to the wall within a certain region of the liquid wedge, let us construct a very simple model. Our principal idea is associated with the events occurring at the TPL. When a vapour bubble is generated on a superheated wall surface and a TPL is formed, the strong evaporation along this line reduces instantly the wall superheat along the TPL practically to zero. The TPL is acting as a circular, radially expanding line heat sink as the bubble grows. The temperature at the TPL is mainly governed by the pressure in the bubble and is expected not to dramatically change during the most period of bubble growth. In the considerations to follow, we will assume such a heat sink to be isothermal and formed along the straight contact line of three bodies, as shown in Fig. 2. For simplicity the TPL should be motionless, whereas the vapour as a third body should not thermally interact with the other two by supposing the corresponding surfaces to be adiabatic. First the whole system is taken to be isothermal.

To simulate thermally the bubble birth, we start the heat sink operation from equilibrium by a jumpwise

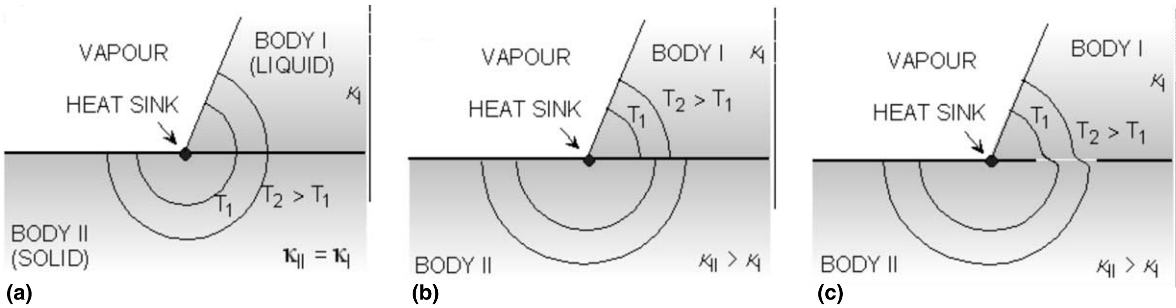

Fig. 2. Illustration of the origin of a heat flux reversal in the course of bubble growth by an isothermal heat sink acting along a TPL. For simplicity, vapour–body surfaces are adiabatic. (a) When the bodies have the same physical properties, temperature waves generated at the heat sink propagate in the bodies at the same velocity and there is no heat flux across the contact surface. (b) Bodies of different physical properties and an adiabatic contact surface. The isotherms are cylindrical surfaces spreading at different velocities in the bodies. (c) Different bodies interacting thermally across the contact surface. The body II is heated by the body I the latter having a lower thermal diffusivity κ_I (e.g., liquid).

reduction of the temperature along the contact line (TPL). Temperature waves thus initiated penetrate the bodies I and II. In Fig. 2(a), the physical properties of these bodies are the same, $\kappa_I = \kappa_{II}$; the temperature waves leaving the heat sink are, therefore, spreading cylindrically outwards at the same velocities. As they arrive at any point of the contact surface of the bodies at the same time, there is no heat flux across this surface, which thus behaves as being adiabatic.

In Fig. 2(b), the bodies have different physical properties ($\kappa_I \neq \kappa_{II}$), but this time their contact surface is assumed to be adiabatic. Also in this case, the isotherms are cylindrical surfaces, which propagate at different velocities, the propagation velocity being faster in the body of the higher thermal diffusivity. Finally, we reject the property of the surface to be adiabatic and allow the bodies I and II to thermally interact with one another. In this case, the isotherms will be shaped as sketched in Fig. 2(c). This shape of the isotherms results in a heat flow between the bodies, the body with the larger thermal diffusivity (solid, κ_{II}) is receiving the heat from the body of the lower diffusivity (e.g., liquid, κ_I). These isotherms agree qualitatively with the ones deduced by Welch [1] from a direct numerical simulation.

This sketch is directly applicable to a growing vapour bubble at a low initial wall superheat. However, in a more detailed analysis the sliding of the TPL, the position of the interface with respect to the wall surface, the initial non-isothermality of the system as well as the fact that the bubble at the start acts practically as a point sink must not be overlooked [6].

4. Concluding remarks

The profile of a wedge-shaped liquid film beneath a growing vapour bubble is governed both by heat convection and by conduction within the liquid phase. The differential equation that describes this profile shows the local change of the film (wedge) thickness to depend on the interfacial liquid velocity, the spatial film slope and the temperature gradient on the liquid side of the interface. The expression for the interfacial heat flux, Eq. (12), derived in this paper, shows this quantity – in agreement with experiments – to increase with increasing boiling temperature (pressure) at otherwise constant conditions. The spatio-temporal development of the temperature field in the immediate surrounding of the TPL illustrates why the wall heat flux may reverse its direction.

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